No. of Printed Pages : 4

Sem-VI-Math-CC-14
(Reg&Back)

2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer
in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 12

(a) Aut(G) is isomorphic to _______ .

(b) Z(G) is a characteristic subgroup of G. (Write True or False.)

(c) Inn(G) = {I} ⇔ G is ________ .

(d) Commutator subgroup is cyclic. (Write True or False.)

(e) In the external direct product group U(8) ⊕ U(10), the product of (3, 7), (7, 9) equals to ________ .

(f) Each orbit is a ________ .

P.T.O.

(g) An action is faithful if its kernel is ________ .

(h) The number of conjugacy classes in D_4 is ________ .

(i) Any finite p-group has ________ centre.

(j) $a \in Z(G) \Leftrightarrow u(a) =$ ________ .

(k) The cyclic group which is isomorphic $Z_2 \oplus Z_3$ is ________ .

(l) The number of conjugacy classes in S_3 is ________ .

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Define outer automorphism of a group.

(b) Find the characteristic subgroup of S_3 .

(c) How many Automorphisms are there in Z_6 ?

(d) Show that $Z_2 \oplus Z_3$ is cyclic.

(e) Define kernel of group action.

(f) State orbit-stabilizer theorem.

(g) Write down the class equation.

(h) State Sylow's 2nd theorem.

(i) State class equation.

(j) State generalized Cayley's theorem.

GROUP – C

3. Answer any eight questions. [3 × 8

(a) Find automorphism of D_3.

(b) Prove that every characteristic subgroup is normal.

(c) Find the order of the element $(2, 7, r^3)$ in the group $Z_3 \times U(10) \times D_4$.

(d) Find all abelian groups of order 56 upto isomorphism.

(e) Let G be a group and $H = \{(g, g) \mid g \in G\}$. Show that H is a subgroup of $G \oplus G$.

(f) Let $H \Delta G$ and let $A = \frac{G}{H}$. Define $* : G \times A \to A$ such that $g * (aH) = g^a g^{-1} H$; $g \in G$, $aH \in A$. Show that $*$ is a group action.

(g) Show that stab(H) = H.

(h) Find the number of 3-Sylow subgroups of order 45.

(i) Find the conjugate classes of S_4 and D_7.

(j) Show that a group of order 16 is not simple.

P.T.O.

GROUP – D

4. Answer <u>any four</u> questions. [7 × 4

(a) Prove that $Aut(Z_n) \approx U(n), \ \forall n \in N$.

(b) State and prove Fundamental theorem of finite abelian group.

(c) State and prove Orbit-Stabilizer theorem.

(d) Prove that A_n is simple, for $n \geq 5$.

(e) State and prove Sylow's 3rd theorem.

(f) State and prove Index theorem.

(g) Find $Aut(Z_{10})$.